TRINITY COLLEGE

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS METHODS UNITS 3&4

If required by your examination administrator, please place your student identification label in this box

Section One: Calculator-free

WA student number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional answer booklets used (if applicable): | |

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-free **35% (50 Marks)**

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

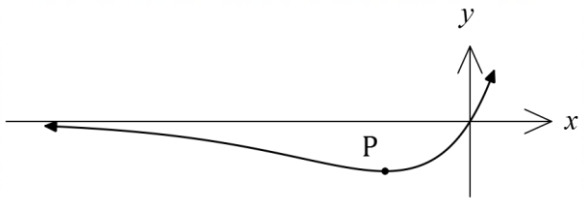
Working time: 50 minutes.

See next page

Question 1 **(7 marks)**

Let $f(x) = 4xe^{(0.25x-1)}$.

The graph of $y = f(x)$ is shown. It has one stationary point, at P, and one point of inflection.



(a) Clearly show that $f'(x) = (x + 4)e^{(0.25x-1)}$. (2 marks)

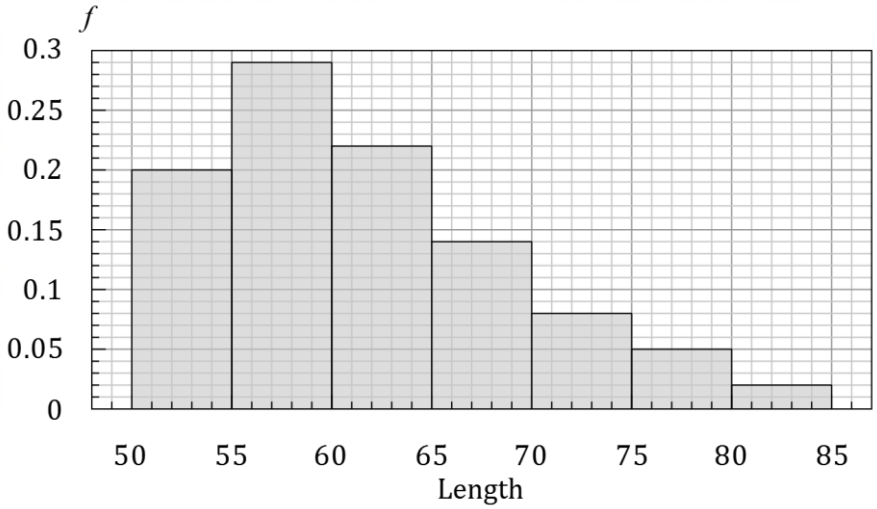
(b) Determine the coordinates of point P. (2 marks)

(c) Determine the values of x for which the curve $y = f(x)$ is concave up. (3 marks)

See next page

Question 2 **(6 marks)**

The relative frequency histogram below shows the distribution of the lengths in centimetres of a large sample of fish bred in an offshore fish farm.



(a) Use the distribution to determine the probability that

(i) a randomly selected fish will be longer than 70 cm. (1 mark)

(ii) a randomly selected fish will be exactly 71 cm long. (1 mark)

(iii) when two fish are randomly selected, one is shorter than 55 cm and the other is not. (2 marks)

(b) An observer claimed that the distribution of the lengths of fish was approximately normal with a mean of 68 cm and standard deviation of 16 cm. Comment on this claim. (2 marks)

See next page

Question 3 **(8 marks)**

Determine the following:

(a) $\int 6e^{2x-3}\,dx$. (1 mark)

(b) $\int_0^{\frac{\pi}{8}} \sin(4x)\,dx$. (2 marks)

(c) $f'\left(\frac{\pi}{6}\right)$ when $f(x) = \frac{\cos(3x)}{2+\sin(x)}$. (3 marks)

(d) $\frac{d}{dx}\int_1^x \cos(t+1)\,dt$. (1 mark)

(e) $\int_0^3 \frac{d}{dx}\left(xe^{2x}\right)dx$. (1 mark)

See next page

Question 4 **(7 marks)**

A computer program scans selected text messages passing through a network to see if the message contains a particular keyword. The random variable X takes the value 0 if the keyword is not found, the value 1 if it is found, and has probability distribution

$$P(X = x) = \begin{cases} \frac{e^{kx}}{3} & x = 0, 1 \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Show that the value of the constant k is $\log_e(2)$. (2 marks)

(b) Determine the mean and standard deviation of X. (2 marks)

(c) Determine the probability that the program finds the keyword in exactly one of the next five randomly selected text messages that it scans. (3 marks)

See next page

Question 5 **(8 marks)**

(a) The velocity, v cm per second, of electrically powered model car C at time t seconds is given by $v = \sqrt{2t + 3}$. Determine the change in displacement of this car between $t = 0.5$ and $t = 6.5$ seconds. (4 marks)

(b) The speed, s cm per second, of model car D at time t seconds is given by $s = e^{\sqrt{2t+3}}$, so that when $t = 3$, its speed was 20.1 cm per second. Use the increments formula to determine a decimal approximation for the speed of this car when $t = 3.05$.

(4 marks)

See next page

Question 6 **(8 marks)**

Components A and B form part of an electronic circuit, and properties of these components are measured t seconds after the circuit is turned on.

(a) The rate of change of temperature, T °C, of component A is given by $\frac{dT}{dt} = \frac{16t}{2t^2 + 5}$.
Determine, in simplest form, the increase in temperature of this component during the first 5 seconds. (4 marks)

(b) The current, I amps, flowing through component B reaches a peak very quickly and then declines as time goes on, as modelled by $I(t) = \frac{3 + \ln(t)}{3t}$. Determine, in simplest form, the maximum current that flows through this component. (4 marks)

See next page

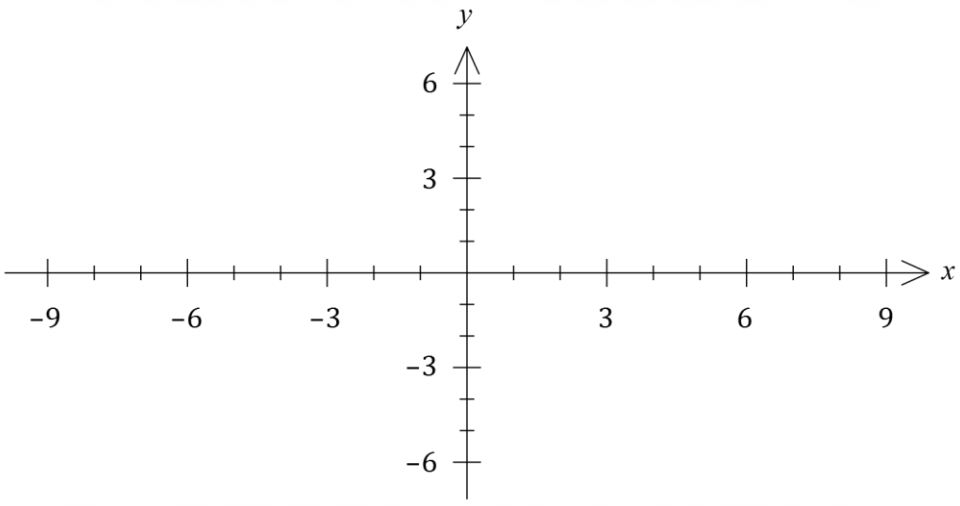
Question 7 **(8 marks)**

Let $f(x) = k\log_5(x+5) + c$, where k and c are constants.

The graph of $y = f(x)$ intersects line L with equation $4y + 3x + 8 = 0$ when $x = 0$ and $x = -4$.

(a) Determine the value of the constant c and the value of the constant k. (3 marks)

(b) Sketch the graph of $y = f(x)$ on the axes below. (3 marks)



(c) Given that $\log_5(x+5) = \dfrac{\ln(x+5)}{\ln(5)}$, determine the value of x where the slopes of $y = f(x)$ and line L are the same. (2 marks)

End of questions

Supplementary page

Question number: _____